MATH 2D Prep: Tangent Lines and Linear Approximation

1. Find an equation of the tangent line to the curve $y = \sin(x)$ at $x = \frac{\pi}{4}$.

Solution:

$$x = \frac{\pi}{4},\ y = \sin(x) = \frac{\sqrt{2}}{2},$$

So the tangent line passes the point $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$.

$$y'(x) = \cos(x),\ y'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},$$

So the slope of the tangent line is $\frac{\sqrt{2}}{2}$.
Give the slope and a point it passes, the equation of the tangent line is

$$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right).$$

2. Estimate $\ln(3)$ without a calculator. You can use the estimation $e \approx 2.7$.

Solution: Given $e \approx 2.7$ and $\ln(x)$ is a continuous function near $x = 3$, we know

$$\ln(3) = \ln(2.7 + 0.3) \approx \ln(e + 0.3).$$

By the formula for linear approximation, and that $[\ln(x)]' = \frac{1}{x}$, we have

$$\ln(e + 0.3) \approx \ln(e) + \frac{1}{e}(e + 0.3 - e) \approx 1 + \frac{0.3}{2.7} = 1 + \frac{1}{9} = 1.111\ldots$$

(Just in case you are curious, the real value is $\ln(3) = 1.0986\ldots$)